Reduced Deadtime and Higher Rate Photon-Counting Detection using a Multiplexed Detector Array

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Abstract

We present a scheme for a photon-counting detection system that can be operated at incident photon rates higher than otherwise possible by suppressing the effects of detector deadtime. The method uses an array of N detectors and a 1-by-N optical switch with a control circuit to direct input light to live detectors. Our calculations and models highlight the advantages of the technique. In particular, using this scheme, a group of N detectors provides an improvement in operation rate that can exceed the improvement that would be obtained by a single detector with deadtime reduced by $1/N$, even if it were feasible to produce a single detector with such a large improvement in deadtime. We model the system for CW and pulsed light sources, both of which are important for quantum metrology and quantum key distribution applications.

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I. INTRODUCTION

There is a long history of low light level measurement applications, such as astronomy and particle physics, with demanding detector requirements. While these applications have provided a steady motivation for detector improvement, the growing interest and advancing efforts in quantum information have brought into sharper focus the need for better photon-counting detectors [1].

Quantum communication and quantum computation applications place difficult design requirements on the manipulation and processing of single photons [2]. Quantum cryptography [3] would particularly benefit from improved detectors, as that application in the form of Quantum Key Distribution (QKD), is currently significantly constrained by detector characteristics such as detection efficiency, dark count rate, timing jitter, and deadtime [4]. Because of demands for higher-rate secret key production, the quantum information community is presently engaged in a number of efforts aimed at improving QKD, including optimizing the quantum channels for minimum loss [5, 6], improving detector efficiency [2, 7, 8], reducing detector timing jitter [9], reducing detector deadtime [10], and single-mode single-photon source engineering [11, 12, 13, 14, 15, 16, 17]. Moreover, with the exponential growth in multimode parametric downconversion (PDC) photon pair production rates now in the range of $2\text{x}10^6$ s^{-1} [18] and the more recent development of $\chi^{(3)}$ single-mode fiber-based sources with pair rates up to 10^7s^{-1} [16, 17], the need is clear for better photon-counting detection by all means possible, including improved deadtime.

In the area of metrology, high-speed detection capability could allow the calibration of a very bright single-photon source, that in the long term could be a viatic for a radiometric standard yielding a "quantum candela"[19]. Additional motivation for this proposal is the improvement of traditional detection applications such as medical diagnosis, bioluminescence, chemical analysis, and material analysis [20, 21, 22, 23].

This idea is an extension of the already established advantage of multiplexing many individual, but imperfect, components into a system that operates with characteristics much closer to the ideal. An example in the field of single-photon technology is the multiplexed single-photon PDC source [24]. In electronics, a more ubiquitous example of this principle would be a computer memory chip or a disk drive where system control bypasses dead or defective subunits. This proposal is also becoming more feasible given the current trends

toward integrated microchip arrays of optical sources and detectors, some of which are now becoming more readily available [20, 21, 22].

We present a scheme that can achieve higher detection rates than is otherwise possible by reducing the effect of detector deadtime. Deadtime is the time needed after a photon-counting detector fires for the detector to recover so that it is ready to register a new photon. This recovery time may be due to the detector, the processing electronics, or some combination of the two. Photomultipliers are a case where the detector deadtime contribution can be quite short, and the subsequent electronics often ultimately set the overall detection deadtime. In avalanche photodiodes (APDs) though, it is more difficult to neatly separate deadtime due to "detection" and deadtime due to "electronics." In an APD, the current avalanche must be physically quenched before the detector is ready for another photon, resulting in a minimum deadtime of typically a few tens of ns. In addition, APDs suffer from afterpulsing that requires an additional wait time before reactivating the detector to avoid a secondary output pulse caused by the previous photon event. In typical APD devices, the resulting deadtimes range from $\approx$50 ns for actively quenched APDs, to $\approx$10 μs for passively quenched APDs, although even actively quenched APDs sometimes employ μs deadtimes to avoid excessive afterpulsing rates. (PMTs also can suffer from afterpulsing, but modern PMTs typically exhibit afterpulsing at much lower rates than APDs [25].)

In practice, detectors are usually operated at detection rates much lower than the inverse of the deadtime to avoid high deadtime fractions and the associated large deadtime corrections. The deadtime fraction (DTF) may be defined as the ratio of missed- to incident-events. Alternately, in the case of a Poissonian CW source, it may be defined as the fraction of the time the detector spends in its recovery state (where it is effectively blind to incoming photons) relative to the total elapsed time. A DTF of 10 % is often a reasonable limit for detector operation. The result is that while many applications would benefit from tens of MHz to GHz detection rates, the reality is that detectors are in practice limited to $\sim$ 1 MHz rates at best. Clearly a way to increase detection rates is needed.

Our scheme to improve detection rates takes a pool of photon-counting detectors and operates them as a unit, or a "detection resource," in a way that allows overall detection at higher rates than would be possible if the detectors were operated individually, while maintaining comparable DTFs. We model and numerically analyze the scheme for typical detector deadtimes, and show the superiority of the scheme over hypothetical single detectors

with much improved deadtimes both for CW- and pulsed-sources. We also note that our scheme bears some resemblance to schemes using beamsplitter trees and detector arrays [26, 27, 28]. We compare the proposed scheme DTFs to those tree schemes, as well as to the performance of a single detector with much reduced deadtime.

II. SCHEME

The detection scheme relies on the rather obvious fact that, while a detector has a significant deadtime when it does fire, it has no deadtime when it does not fire. The scheme consists of a 1-by-N optical switch that takes a single input stream of photons and distributes them to members of an array of N detectors. A switch control circuit monitors which detectors have fired recently and are thus dead, and then routes subsequent incoming pulses to a detector that is ready. This system allows a system of N detectors to be operated at a significantly higher detection rate than N times the detection rate of an individual detector, while maintaining the same DTF.

To understand the process, consider a fixed input photon pulse rate, some pulses of which may contain a photon and some may not. (For example, this is usually the situation in a quantum cryptography application.) At the start of operation, all detectors are live and ready to detect a photon. The switch is set to direct the first incoming pulse to the first detector of the array. Control electronics monitor the output of that detector to determine if it fires. If the detector does fire, the control switches the next pulse to the next detector. If the detector does not fire, then the switch state remains unchanged. The process repeats with the input always directed to the first available live detector. At high count rates many of the detectors may fire in a short period of time and subsequently be in their dead state, but as long at the first detector recovers to its live state before the last detector triggers, the system will still be live and ready to register an incoming photon. The system will only be dead when all detectors have fired within one deadtime of each other. The system operation could be sequential with each detector firing in order as just described, or it could be set up to direct the input to any live detector. That would allow for optimum use of an array of detectors where each detector may have a different deadtime.

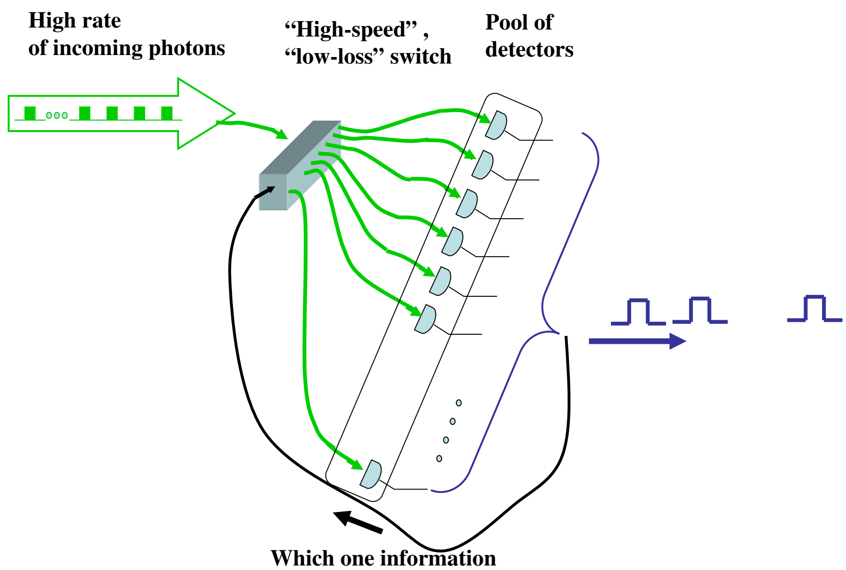

FIG. 1: A pool of detectors and a fast switch are used to register a high rate of incoming photons. Incoming photons are switched to a ready detector. If it fires, the detector is switched out of the ready pool until recovery. If it does not fire, that detector remains ready.

III. ANALYSIS AND MODELS

To understand the system operation and quantify its advantages relative to non-multiplexed systems, we use approximate analytical and numerical Monte Carlo models, both for the cases of a CW Poisson distributed- and a pulsed-source.

A. CW (Poisson) source: analytical modeling

Our analytical calculation estimates the DTF from the mean total count rate to the overall detector pool and effective deadtimes for each detector (which depend on their position in the switching system). We consider a Poissonian source and a pool of detectors with the identical detection efficiencies η and identical non-extending deadtimes T_{d} [29]. Zero switch transition time is assumed. (We refer to a Poissonian source as CW because, while the photons arrive at discrete times, they have equal probability to arrive at any time.) Furthermore, the optical switch is programmed to send photons to the detectors in sequence. The switch

sends photons always to detector 1 (D_1) when it is live. If it is dead, it sends the photons to D_2, and so on.

The probability that n photons from a Poissonian source with mean photon rate λ are registered by a single live detector with efficiency η in a time interval T is $P(n) = (\eta\lambda T)^n e^{-\eta\lambda T}/n!$. Thus, the mean number of counts registered is $\eta\lambda T$. From here on for simplicity we assume $\eta = 1$. (We can do this without loss of generality, as η and λ always appear together and can thus be traded off against each other with affecting the ultimate results.) In the presence of deadtime T_d and for measurement time $T \gg T_\mathrm{d}$, the mean number of counts registered reduces to

$$M = \lambda T - M\lambda T_\mathrm{d}, \tag{1}$$

where $M\lambda T_\mathrm{d}$ accounts for the mean number of photons lost to deadtime. Rearranging, we have

$$M = \frac{\lambda T}{1+\lambda T_\mathrm{d}}. \tag{2}$$

The DTF, defined as the ratio of the lost counts over the total counts in the absence of dead time, for this simple case is

$$\mathrm{DTF} = \frac{\lambda T - M}{\lambda T} = 1 - \frac{1}{1+\lambda T_\mathrm{d}}. \tag{3}$$

Now consider an array of detectors connected to an optical switch (Fig. 1) with switching time negligible with respect to T_d. Eq. (2) holds for D_1, so the mean number of counts detected by D_1 is $M_1 = \frac{\lambda T}{1+\lambda T_\mathrm{d}}$, while D_1 is dead during a time interval $T_2 = M_1 T_\mathrm{d}$. Thus the time interval during which D_2 may count photons is T_2 [31]. Continuing the analogy with Eq. (2) the mean number of counts detected by D_2 is

$$M_2 = \frac{\lambda T_2}{1+\lambda \mathcal{T}_{\mathrm{d}(2)}}, \tag{4}$$

where here $\mathcal{T}_{\mathrm{d}(2)}$ is the effective deadtime associated with D_2. It is necessary to introduce an effective deadtime because the measurement time T_2 is not reduced by the full deadtime T_d. Only part of the deadtime of D_2 will occur while D_1 is dead, effectively reducing $\mathcal{T}_{\mathrm{d}(2)}$. We postpone the evaluation of effective deadtimes.

In analogy with the arguments leading to Eq. (2), D_3 is live during the time interval $T_3 = M_2\mathcal{T}_{\mathrm{d}(2)}$, corresponding to the time interval when both D_1 and D_2 are dead, and the

mean number of counts registered by D_3 is

$$M_3 = \frac{\lambda T_3}{1 + \lambda \mathcal{T}_{d(3)}}, \tag{5}$$

where $\mathcal{T}_{d(3)}$ is the effective deadtime associated with D_3. Likewise for detector D_i, the measurement time is $T_i = M_{i-1}\mathcal{T}_{d(i-1)}$ and the mean count rate is $M_i = \frac{\lambda T_i}{1+\lambda \mathcal{T}_{d(i)}}$.

The mean number of counts registered by the multiplexed detector system with N-detectors is $M_{tot} = M_1 + M_2 + ... + M_N$, and the overall system DTF $= \frac{\lambda T - M_{tot}}{\lambda T}$ is

$$\mathrm{DTF} \ = \ 1 - \frac{1}{1+\lambda T_d}\left(\begin{array}{c} 1 + \frac{\lambda T_d}{1+\lambda \mathcal{T}_{d(2)}} + \frac{\lambda^2 T_d \mathcal{T}_{d(2)}}{(1+\lambda \mathcal{T}_{d(2)})(1+\lambda \mathcal{T}_{d(3)})} + ... \\ + \frac{\lambda^{N-1} T_d \mathcal{T}_{d(2)} ... \mathcal{T}_{d(N-1)}}{(1+\lambda \mathcal{T}_{d(2)})(1+\lambda \mathcal{T}_{d(3)})...(1+\lambda \mathcal{T}_{d(N)})} \end{array}\right). \tag{6}$$

For comparison, as we will see in our subsequent analysis and modeling, the DTF obtainable by simply reducing T_d of a single detector by a factor of $1/N$ is DTF$=1 - \frac{1}{1+\lambda T_d/N}$. We note also that this result is the same as would be obtained by an array of N detectors with deadtime T_d and passive switching such as may be implemented with a tree arrangement of beam splitters. This result follows from the fact that, in such a tree, the incident rate at each detector is λ/N.

We analyse the effective deadtime of D_2 for two cases using Fig. 2. The top timeline indicates the arrival times of photons. The 2nd and 3rd timelines indicate when D_1 and D_2 register counts and when they are dead (shaded regions). $\Delta = t_3 - t_1$ is the time interval between the first photon counted by D_1 and the first one after its deadtime T_d has expired. $\delta = t_2 - t_1$ is the time interval between the first two photon arrivals at times t_1 and t_2, the first detected by D_1 and the second by D_2. Fig. 2 shows two possible situations. In case (a), the time interval between two sequential counts of D_1 (t_1, t_3) is larger than the time interval between the count at t_1 and the count at t_2 plus T_d the deadtime of D_1, namely $\Delta > \delta + T_d$. In this case the effective deadtime of the detector combination of D_1 and D_2, $\mathcal{T}_{d(2)}$, is $T_d - \delta$. In case (b) the time interval Δ is shorter than $\delta + T_d$, thus two terms contribute to the effective deadtime, $\mathcal{T}_1 = T_d - \delta$ and $\mathcal{T}_2 = T_d + \delta - \Delta$.

As we assumed that the arrival of photons at the array of detectors is Poissonian, the random variables Δ and δ are statistically independent. The probability density function of the random variable Δ is $f_\Delta(\Delta) = \lambda e^{-\lambda(\Delta - T_d)}\Theta(\Delta - T_d)$, where $\Theta(x) = 1$ for $x > 0$, and 0 otherwise. The probability density function of the random variable δ is $f_\delta(\delta) = \lambda e^{-\lambda\delta}/(1 - e^{-\lambda T_d})\Theta(T_d - \delta)$.

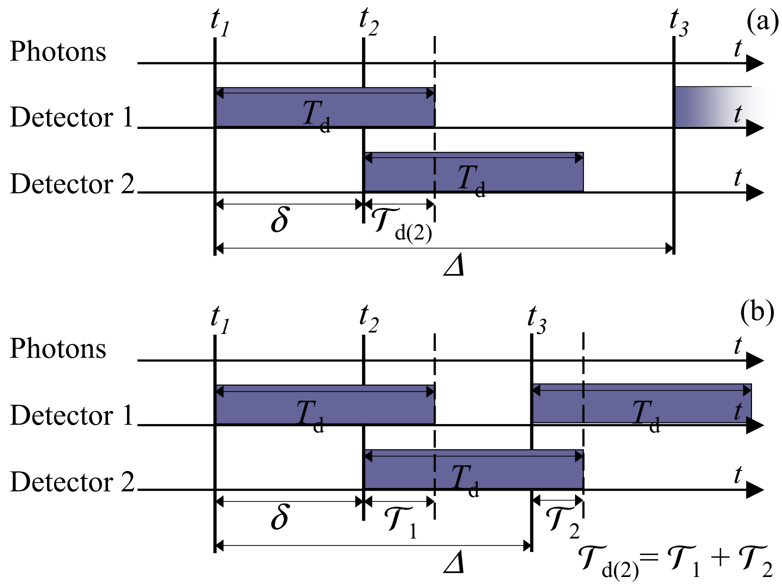

FIG. 2: The effective deadtime of D_1. Case (a) the time interval between two subsequent counts of D_1, $\Delta > \delta + T_d$,with δ the time interval between the count of D_1 and the subsequent photon counted by D_2 during the deadtime of D_1, and T_d the single detector deadtime. Case (b) $\Delta < \delta + T_d$. In this second case two terms, $\mathcal{T}_1$ and $\mathcal{T}_2$ contribute to the final deadtime. Dark shaded regions represent the deadtime of the individual detectors

The probability that situation (a) occurs is

$$p_a = \int_{\Delta > \delta + T_d} f_\Delta(\Delta) f_\delta(\delta) \mathrm{d}\Delta \mathrm{d}\delta, \tag{7}$$

while the probability that situation (b) occurs is

$$p_b = \int_{\Delta < \delta + T_d} f_\Delta(\Delta) f_\delta(\delta) \mathrm{d}\Delta \mathrm{d}\delta. \tag{8}$$

In case (a), the mean value of the "effective" deadtime is $\mathcal{T}_a = T_d - \mathrm{E}_a(\delta)$ with

$$\mathrm{E}_a(\delta) = \frac{\int_{\Delta > \delta + T_d} \delta f_\Delta(\Delta) f_\delta(\delta) \mathrm{d}\Delta \mathrm{d}\delta}{\int_{\Delta > \delta + T_d} f_\Delta(\Delta) f_\delta(\delta) \mathrm{d}\Delta \mathrm{d}\delta}. \tag{9}$$

In case (b), the mean value of the "effective" deadtime is $\mathcal{T}_b = 2T_d - \mathrm{E}_b(\Delta)$ with

$$\mathrm{E}_b(\Delta) = \frac{\int_{\Delta < \delta + T_d} \Delta f_\Delta(\Delta) f_\delta(\delta) \mathrm{d}\Delta \mathrm{d}\delta}{\int_{\Delta < \delta + T_d} f_\Delta(\Delta) f_\delta(\delta) \mathrm{d}\Delta \mathrm{d}\delta}. \tag{10}$$

The mean effective deadtime $\mathcal{T}_{\mathrm{d}(2)} = p_{\mathrm{a}}\mathcal{T}_{\mathrm{a}} + p_{\mathrm{b}}\mathcal{T}_{\mathrm{b}}$ can be calculated as

$$\mathcal{T}_{\mathrm{d}(2)} = T_{\mathrm{d}} - \frac{1 - e^{-\lambda T_{\mathrm{d}}}}{2\lambda}. \tag{11}$$

We iterate the formula for the subsequent detectors obtaining a recursive expression for the effective final deadtime, $\mathcal{T}_{\mathrm{d}(i)} = \mathcal{T}_{\mathrm{d}(i-1)} - \frac{1-e^{-\lambda \mathcal{T}_{\mathrm{d}(i-1)}}}{2\lambda}$. The calculated results for DTF versus incident photon rate for pools of up to 12 detectors are nearly identical to the Monte Carlo results shown in Fig. 3 and described in the next section.

B. CW Source: Monte Carlo Modeling

The Monte Carlo model assumes a CW source with Poisson distributed incident photons at a range of rates meant to describe the use of the system in conjunction with a laser source. As mentioned before, we can assume 100 % efficient collection and detection without loss of generality. The individual detector deadtimes were set to 50 ns. The modeling procedure consisted of first using a random number generator to simulate an input stream of a large number of photons with Poisson distributed arrival times. The resulting photon list was then apportioned to the first detector by going through each photon time on the list in sequence to see if it could have been detected by a single detector D_1. That is, once a photon is detected, any photons within one deadtime after that detected photon are skipped. A new list consisting of the "skipped" photons was then apportioned to the second detector D_2 using the same procedure as for the first detector. This process was repeated for all N detectors. Those photons left after detector D_N are those that would be missed by the system and the ratio to the total number in the original photon list is the deadtime fraction, as previously defined.

Figure 3a shows the resulting DTF versus mean incident photon rate for systems of varying numbers of detectors. The Monte Carlo results (shown) and the analytical calculations (not shown) provide nearly identical results. From the $R_{\mathrm{DTF}=10\%}$ points (defined as the incident photon rate at which the DTF= 10 %), we see that for example, a system of 6 detectors can operate at 32 times the incident rate of a single detector while maintaining 10 % deadtime. This is significantly more than just six times the single detector count rate, the improvement possible with a passive switch arrangement, highlighting the power of the technique. We also see that the multiplexed detector scheme also has an advantage over

simply reducing the deadtime of an individual detector. Even reducing the deadtime by a factor of 10 (to 5 ns) does not allow improvements equal to the system with 6 detectors with $T_{\mathrm{d}} = 50$ ns.

Figure 3b shows the dependence of $R_{\mathrm{DTF}=10\%}$ on the number of detectors in the system, which fits well to a 2nd order polynomial. The origin of this behavior is due to the correction in the effective deadtime of each detector of the pool, embedding a nonlinear dependence on the number of detectors, while the same behavior is not present for a passive switch (or a tree of beamsplitters), as shown by its linear $R_{\mathrm{DTF}=10\%}$ dependence on the number of detectors.

For complete modeling of this scheme other switch parameters such as switch losses, switching transition times, switch latency, maximum switching rates, and cross-talk should be included. Of these, switch loss is probably the most problematic, as commercially available ns switches have losses ≈2-3 dB, although there are ongoing efforts to address this issue. Loss affects the overall detection efficiency so it should not affect the functional behavior of the results presented here. Switch transition and latency times should have effects similar to increasing the deadtime of the individual detectors as well as reducing overall detection efficiency. However, with some commercial switch transition times being below 50 ps, that should not be a severe limit. These parameters will be the subject of further modeling as they will be what ultimately limits how far this method can be pushed.

C. Pulsed source: analytical modeling

The detection scheme (Fig. 1) modeling with a pulsed source proceeds in similar fashion to the CW case, and the formulas derived below are analogous to the CW results. Each pulse of the source may contain 0, 1 or more photons, but because the detector we are modeling cannot distinguish between 1 or more photons it has only two output possibilities: it either fires or it does not. The probability that a live detector produces a count ("event") for an individual pulse is p. The probability that n events are counted by a single live detector in a sequence of $\mathcal{N} = \nu T$ pulses (where ν is the repetition rate of the pulsed source, and T the measurement time) is $B(n|\mathcal{N}, p) = \mathcal{N}![n!(\mathcal{N} - n)!]^{-1} p^n (1 - p)^{\mathcal{N}-n}$. From this, it can be shown that the mean number of counts is $p\mathcal{N}$ and the mean count rate is $p\nu$. In the presence of deadtime, the detector is dead for a certain number of pulses $\mathcal{N}_{\mathrm{d}} = \mathrm{Int}(\nu T_{\mathrm{d}})$, where T_{d}

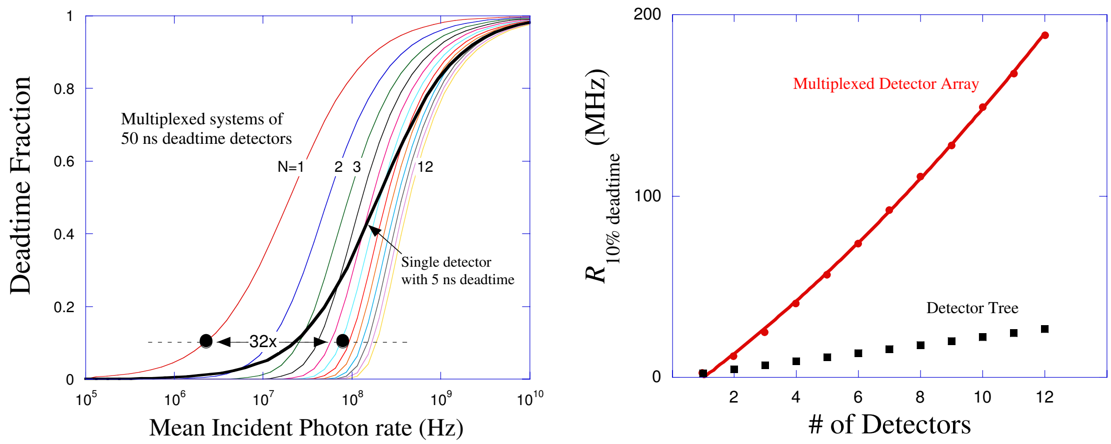

FIG. 3: a.) DTF versus mean incident photon rate shown for systems of 1 to 12 detectors, all with 50 ns deadtimes. The dashed horizontal line shows the 10 % DTF level that is often the practical limit for detector operation. The DTF for a single detector with 5 ns deadtime is shown for comparison (thick line). As an example of the advantage of the scheme, the two points illustrating that with 6 detectors the system would be able to operate at a 32 times larger incident photon rate than a single detector, while maintaining that 10 % DTF. b.) $R_{\mathrm{DTF}=10\%}$ (circles) versus number of detectors in the detector pool, along with a fit to a 2nd order polynomial (line). For comparison the dependence of $R_{\mathrm{DTF}=10\%}$ for a detector tree is also shown (squares).

is the deadtime of the single detector and Int indicates the integer part. For measurement time such that $\mathcal{N} \gg \mathcal{N}_{\mathrm{d}}$, the mean number of counts reduces to

$$M = p\mathcal{N} - M\ p\mathcal{N}_{\mathrm{d}}, \tag{12}$$

where $p\mathcal{N}_{\mathrm{d}}$ is the mean number of events lost during one deadtime. Thus, the mean number of counted events is

$$M = \frac{p\mathcal{N}}{1 + p\mathcal{N}_{\mathrm{d}}}. \tag{13}$$

Eq. (13) holds for the first detector D_1, so the mean number of counts detected by D_1 is $M_1 = \frac{p\mathcal{N}}{1+p\mathcal{N}_{\mathrm{d}}}$, while D_1 is dead for the average number of pulses in the measurement time $\mathcal{N}_2 = M_1\mathcal{N}_{\mathrm{d}}$. Thus, the time interval during which D_2 may count photons is $\mathcal{N}_2$. Continuing the analogy with the CW case, the mean number of counts detected by D_2 is

$$M_2 = \frac{p\mathcal{N}_2}{1 + p\mathcal{N}_{\mathrm{d}(2)}}, \tag{14}$$

where here $\mathcal{N}_{\mathrm{d}(2)}$ is the mean number of pulses constituting this "effective deadtime" associated to D_2. The evaluation of the "effective deadtimes" is presented below. Thus, for D_i, the average number of pulses in the measurement time is $\mathcal{N}_{(i)} = M_{i-1}\mathcal{N}_{\mathrm{d}(i-1)}$, and the mean count rate is $M_i = \frac{p\mathcal{N}_i}{1+p\mathcal{N}_{\mathrm{d}(i)}}$.

The mean number of counts registered by the multiplexed detector system is $M_{\mathrm{tot}} = M_1 + M_2 + ... + M_N$, and the DTF$= \frac{p\mathcal{N} - M_{\mathrm{tot}}}{p\mathcal{N}}$ is

$$\mathrm{DTF} = 1 - \frac{1}{1+p\mathcal{N}_{\mathrm{d}}} \left(\begin{array}{c} 1 + \frac{p\mathcal{N}_{\mathrm{d}}}{1+p\mathcal{N}_{\mathrm{d}(2)}} + \frac{p^2\mathcal{N}_{\mathrm{d}}\mathcal{N}_{\mathrm{d}(2)}}{(1+p\mathcal{N}_{\mathrm{d}(2)})(1+p\mathcal{N}_{\mathrm{d}(3)})} + ... \\ + \frac{p^{N-1}\mathcal{N}_{\mathrm{d}}\mathcal{N}_{\mathrm{d}(2)}...\mathcal{N}_{\mathrm{d}(N-1)}}{(1+p\mathcal{N}_{\mathrm{d}(2)})(1+p\mathcal{N}_{\mathrm{d}(3)})...(1+p\mathcal{N}_{\mathrm{d}(N)})} \end{array} \right).$$

We note that if the number of the detectors in the multiplexed array is more than $\mathcal{N}_{\mathrm{d}} + 1$, all the events will be detected and DTF will be always zero.

In the pulsed case, the advantage obtainable with a single detector with deadtime reduced of a factor $1/N$ is given by DTF$= 1 - \frac{1}{1+p\ \mathrm{Int}(\nu T_{\mathrm{d}}/N)}$, while for the detector tree configuration the deadtime fraction is DTF$= 1 - \frac{1}{1+p\mathcal{N}_{\mathrm{d}}/N}$. Note that for the case of a single detector with reduced deadtime, when $T_{\mathrm{d}}/N < 1/\nu$ then the DTF=0, while this is never the case for the detector tree configuration.

We analyze the "effective" deadtime of D_2 for a pulsed source using Fig. 4 where the dashed vertical lines represent empty pulses and the continuous vertical lines represent detection events. As with Fig. 2, the D_1 and D_2 timelines indicate when D_1 and D_2 register a count and when they are dead (dark shaded regions). n_Δ is the number of pulses between the first photon counted by D_1 and the subsequent one, after its deadtime $\mathcal{N}_{\mathrm{d}}$. n_δ is the number of pulses between the first two events, the first detected by D_1 and the second by D_2. Fig. 5 shows two possible situations. In case (a), the time interval between two subsequent counts of D_1 is larger than the time interval between the first detected by D_1 and the second by D_2 (during the deadtime of D_1) plus the deadtime of the D_2, namely $n_\Delta \geq n_\delta + \mathcal{N}_{\mathrm{d}}$. In this case, the effective deadtime of the detector combination of D_1 and D_2, $\mathcal{N}_{\mathrm{d}(2)}$, is $\mathcal{N}_{\mathrm{d}} - n_\delta$. In case (b) the time interval n_Δ is shorter then $n_\delta + \mathcal{N}_{\mathrm{d}}$, thus two terms contribute to the effective deadtime, the effective deadtime, $\mathcal{P}_1 = \mathcal{N}_{\mathrm{d}} - n_\delta$ and $\mathcal{P}_2 = \mathcal{N}_{\mathrm{d}} + n_\delta - n_\Delta$.

We consider the random variables n_Δ and n_δ to be statistically independent as there is no correlation between pulses. The probability distribution of n_Δ is $P_\Delta(n_\Delta) = p(1-p)^{n_\Delta - \mathcal{N}_{\mathrm{d}} - 1}$, with n_Δ integer and $n_\Delta \geq \mathcal{N}_{\mathrm{d}}+1$. The n_δ probability distribution is $P_\delta(n_\delta) = p(1-p)^{n_\delta - 1}[1-(1-p)^{\mathcal{N}_{\mathrm{d}}}]^{-1}$, with n_δ integer and $1 \leq n_\delta \leq \mathcal{N}_{\mathrm{d}}$.

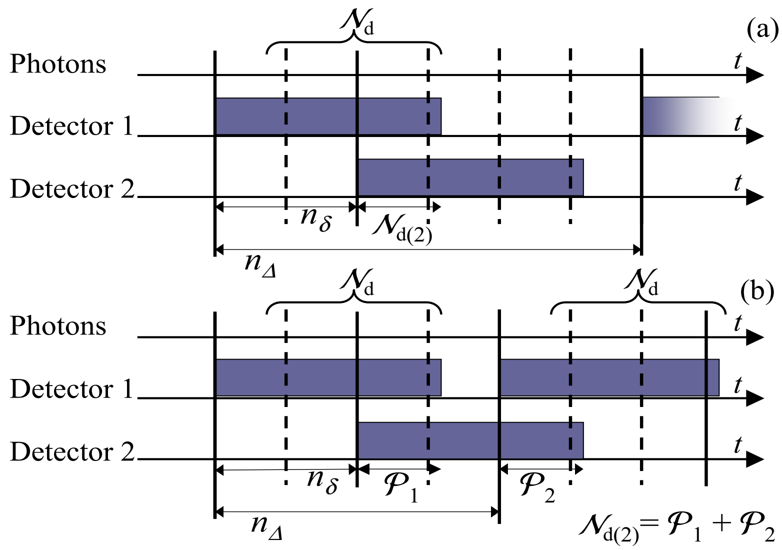

FIG. 4: The effective deadtime of D_1. Case (a) the time interval between two subsequent counts of D_1, $n_\Delta \geq n_\delta + \mathcal{N}_{\rm d}$,with n_δ the time interval between the count of D_1 and the subsequent photon counted by D_2 during the deadtime of D_1, and $T_{\rm d}$ the actual single detector deadtime. Case (b) $n_\Delta < n_\delta + \mathcal{N}_{\rm d}$. In this second, case two terms, $\mathcal{P}_1$ and $\mathcal{P}_2$ contribute to the final deadtime.

The probability that situation (a) ($n_\Delta \geq n_\delta + \mathcal{N}_{\rm d}$) occurs is

$$p_a = \sum_{n_\delta=1}^{\mathcal{N}_{\rm d}} \sum_{n_\Delta = n_\delta + \mathcal{N}_{\rm d}}^{+\infty} P_\Delta(n_\Delta) P_\delta(n_\delta), \tag{15}$$

while the probability that situation (b) ($n_\Delta < n_\delta + \mathcal{N}_{\rm d}$) occurs is

$$p_b = \sum_{n_\delta=2}^{\mathcal{N}_{\rm d}} \sum_{n_\Delta = \mathcal{N}_{\rm d}+1}^{n_\delta + \mathcal{N}_{\rm d} - 1} P_\Delta(n_\Delta) P_\delta(n_\delta). \tag{16}$$

In case (a) the mean value of the "effective" the deadtime is $\mathcal{N}_{\rm d,a} = \mathcal{N}_{\rm d} - E_{\rm a}(n_\delta)$, with

$$E_{\rm a}(n_\delta) = \frac{\sum_{n_\delta=1}^{\mathcal{N}_{\rm d}} \sum_{n_\Delta = n_\delta + \mathcal{N}_{\rm d}}^{+\infty} n_\delta P_\Delta(n_\Delta) P_\delta(n_\delta)}{\sum_{n_\delta=1}^{\mathcal{N}_{\rm d}} \sum_{n_\Delta = n_\delta + \mathcal{N}_{\rm d}}^{+\infty} P_\Delta(n_\Delta) P_\delta(n_\delta)}. \tag{17}$$

While in case (b) the mean value of the "effective" deadtime is $\mathcal{N}_{\rm d,b} = 2\mathcal{N}_{\rm d} - E_{\rm b}(n_\Delta)$, with

$$E_{\rm b}(n_\Delta) = \frac{\sum_{n_\delta=2}^{\mathcal{N}_{\rm d}} \sum_{n_\Delta = \mathcal{N}_{\rm d}}^{n_\delta + \mathcal{N}_{\rm d} - 1} n_\Delta P_\Delta(n_\Delta) P_\delta(n_\delta)}{\sum_{n_\delta=2}^{\mathcal{N}_{\rm d}} \sum_{n_\Delta = \mathcal{N}_{\rm d}}^{n_\delta = \mathcal{N}_{\rm d} - 1} P_\Delta(n_\Delta) P_\delta(n_\delta)}. \tag{18}$$

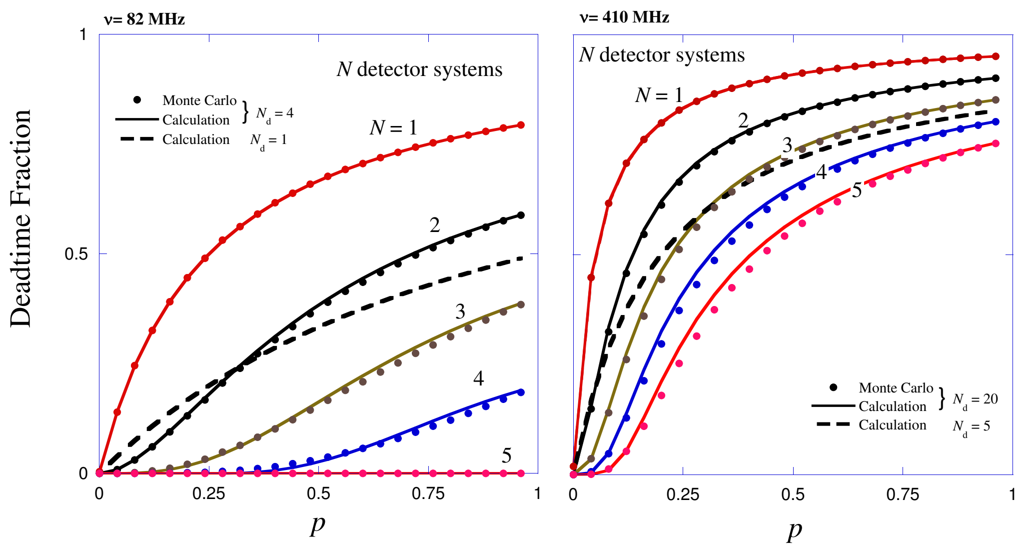

FIG. 5: Deadtime Fraction versus probability of detection per pulse for pools of up to 5 detectors, for detectors each with $T_\mathrm{d} = 50$ ns, with a pulsed source with repetition rate of (a) 82 MHz ($\mathcal{N}_\mathrm{d} = 4$), and (b) 410 MHz ($\mathcal{N}_\mathrm{d} = 20$). Circles represent the Monte Carlo simulations, while solid lines are the calculated values according to the here presented theory. For comparison, a single-detector-DTF (broken line) with a reduced deadtime of 12.5 ns, corresponding to (a) $\mathcal{N}_\mathrm{d} = 1$ and (b) $\mathcal{N}_\mathrm{d} = 5$ are shown.

The mean effective deadtime $\mathcal{N}_{\mathrm{d}(2)} = p_\mathrm{a}\mathcal{N}_\mathrm{d,a} + p_\mathrm{b}\mathcal{N}_\mathrm{d,b}$ can be calculated as

$$\mathcal{N}_{\mathrm{d}(2)} = \mathcal{N}_\mathrm{d} - \frac{1-(1-p)^{\mathcal{N}_\mathrm{d}+1}}{(2-p)p}. \tag{19}$$

We iterate the formula for the following detectors obtaining a recursive expression for the effective final deadtime, $\mathcal{N}_{\mathrm{d}(i)} = \mathcal{N}_{\mathrm{d}(i-1)} - \frac{1-(1-p)^{\mathcal{N}_{\mathrm{d}(i-1)}+1}}{(2-p)p}$. The calculated results for DTF versus incident photon rate for pools of up to 5 detectors are shown in Fig. 5. Monte Carlo results as described in the next section are also shown.

D. Pulsed Source: Monte Carlo Modeling

The Monte Carlo model assumes a geometric distribution which samples pulse arrival times as shown in Appendix A. As mentioned before, the detectors cannot discriminate between one or more photons in a single pulse and can fire at most once during a pulse, so the probability of detecting an event per pulse spans from 0 to 1. As in the CW case, a random number generator was used to simulate the source, although this time it is a pulsed source with geometric distributed events. As before, the resulting event list is apportioned to each detector in sequence to see if it could have been detected or if it is skipped by that detector. Those events left after the N^{th} detector are those that would be missed by the system and the ratio to the total number in the original event list is the deadtime fraction.

In Fig. 5 we show the DTF versus the probability of detection of an event per pulse p, for pools of up to 5 detectors each with 50 ns deadtime, for a pulsed source with repetition rate of (a) 82 MHz ($\mathcal{N}_\mathrm{d} = 4$) and of (b) 410 MHz ($\mathcal{N}_\mathrm{d} = 20$). In Fig. 6 (a) we observe that, as expected, the DTF is 0 in the case of a pool of five or more detectors.

To highlight the advantage of the multiplexed detector scheme, we compare the performance of the multiplexed detector system to a single detector with 4x reduced deadtime of 12.5 ns. Fig. 6, also shows a comparison of $R_{DTF=10\%}$, for a pulsed source with repetition rates of 82 MHz and 410 MHz, for a single detector with reduced dead time, for the detector tree configuration and our scheme. Because $R_{DTF=10\%}$ increases quadratically with the number of detectors, the multiplexed configuration, even with just a few detectors, provides better performance than the other configurations. In fact, only when the deadtime of single detector is made shorter than the pulse separation time, can it achieve the same performance as the multiplexed scheme.

IV. CONCLUSION

We have shown that a pool of N detectors with a controlled switch system can in principle be operated at much higher incident photon rates than is otherwise possible either with a single detector with much reduced deadtime, or an array of detectors with passive switch system such as might be implemented with by a tree of beamsplitters [32]. This advantage holds for both CW and pulsed sources. We note from a practical view, that a multiplexed

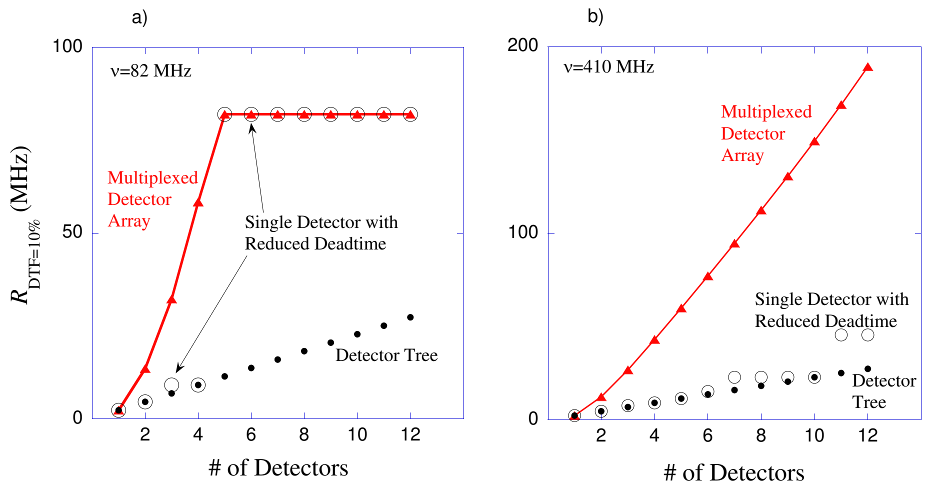

FIG. 6: $R_{DTF=10\%}$ (the incident rate at which the DTF reaches 10%) of a multiplexed detector array (triangles), a single detector with reduced deadtime (open circles), and a detector tree configuration (solid circles) for pulsed sources with repetition rates of (a) 82 MHz, (b) 410 MHz. The single detector points in (a) and (b) are plotted versus the $1/N$ reduction in $T_{\rm d}$.

system may be easier to implement for a pulsed source as the switching time need only be smaller than the time separation between pulses. We also note that two factors are working to increase the relevance of this scheme - a) advancing quantum information applications are increasing the need for higher performance detectors, and b) improving array detectors and low-loss high-speed switches are making this scheme more practical. Moreover our scheme could also be implemented with photon-number-resolving (PNR) detectors, as well as the non-photon-number-resolving detectors typically used for "photon counting" and analyzed in this work. The advantage of reduced deadtime, in combination with a PNR detector array, would make for a very powerful detection capability indeed.

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APPENDIX A: PULSED PROCESS AND GEOMETRIC DISTRIBUTION

In analogy with Ref. [30], where the connection between the Poissonian process and the Poissonian probability distribution are described, we describe the connection between the pulsed process and the geometric distribution. Consider a pulsed process, with probability p of detecting an event for each pulse. The probability of waiting n pulses before detecting an event is given by the geometric probability distribution

$$\mathcal{T}(n) = p(1-p)^{n-1}.$$

The probability of waiting n pulses before detecting two events (meaning that the second event is detected at the n^{th} pulse) is

$$\mathcal{T}_2(n) = pB(1|n-1, p),$$

and analogous arguments hold for the probability of waiting n pulses before detecting three, four, etc... events. Thus in general the probability of waiting n pulses before detecting $k-1$ events is given by the generalized geometric probability

$$\mathcal{T}_k(n) = pB(k-1|n-1, p).$$

Thus the probability that there are more than k events in $\mathcal{N}$ pulses is $P(m \geq k \text{ int } \mathcal{N}) = \sum_{n=k}^{\mathcal{N}} \mathcal{T}_k(n)$. The probability of exactly k events in $\mathcal{N}$ pulses obviously is

$$P(m \geq k \text{ in } \mathcal{N}) - P(m \geq k+1 \text{ in } \mathcal{N}) = \sum_{n=k}^{\mathcal{N}} \mathcal{T}_k(n) - \sum_{n=k+1}^{\mathcal{N}} \mathcal{T}_{k+1}(n).$$

Ultimately we see that $\sum_{n=k}^{\mathcal{N}} \mathcal{T}_k(n) - \sum_{n=k+1}^{\mathcal{N}} \mathcal{T}_{k+1}(n) = B(k|\mathcal{N}, p)$.

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[32] This latter result is not too surprising as the schemes using beamsplitter trees and detector arrays were created to overcome the lack of PNR capability of most photon-counting detectors [26, 27, 28]. In other words, they are designed to solve the problem of photons arriving at exactly the same time rather than just arriving very close in time. While our scheme enhances $R_{\mathrm{DTF}=10\%}$, it provides no PNR capability.